A) Vertical projectiles:
We have actually done these before. Let's use the baseball example. A batter hits a high pop-fly that rises straight up from home plate with an initial velocity of 35 m/s.

Now, the question probably says something else, but you can pretty much stop reading b/c you know this is a projectile and therefore you have three questions to answer:

1) How high?
2) How far?
3) How much time?

2) This is easy. Zero meters, if the ball goes straight up, it comes straight down.
1 & 3) These are related and the order you answer them is irrelevant. There are multiple ways to solve this problem. Here is one way:

This problem is y-direction only

$x = ?$ (How high?)

$t = ?$ (How much time?)

$V_0 = 35 m/s$ up

$V = 0$ @ the top

$a = 9.81 m/s^2$ down

to find time

$$a = \frac{\Delta V}{t}$$

$$-9.81 = \frac{0 - 35}{t}$$

$t_{up} = 3.57 s$

$t_{tot} = 7.14 s$

to find height

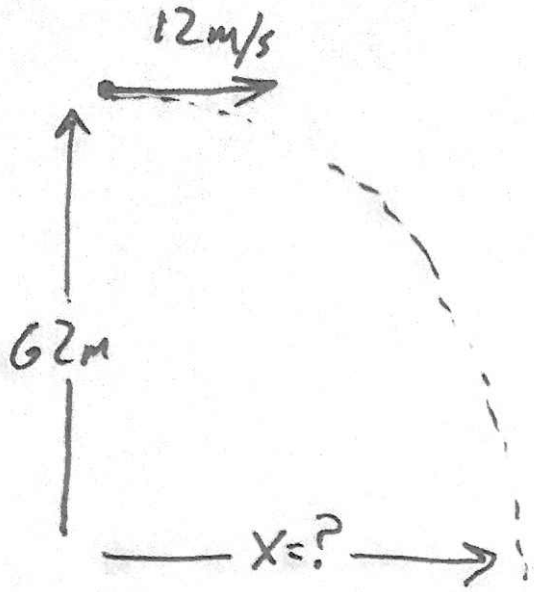
$$x = v_0 t + \frac{1}{2} a t^2$$

use t_{up} $\longrightarrow$ $y = (35)(3.57) + \frac{1}{2}(-9.81)(3.57^2)$

$y = 62.4 m$

B) Horizontal Projectiles: We call this type of problem a horizontal projectile because the initial velocity vector is ONLY in the horizontal direction. Due to the influence of gravity, this projectile begins falling just as soon as it is released, but continues to move horizontally at the same speed it was traveling, since the only force is gravity, in the y-direction. The bullet fired from the gun in the earlier example is a horizontal projectile. Here is another example:

A boy throws a rock off the side of a 62 m tall cliff. If the boy pitches the rock horizontally at 12 m/s…now you have 3 questions: How high? How far? How much time?



x

Constant velocity

$V = \frac{\Delta x}{t}$

$x = ?$

$V = 12 m/s$

$t = ?$

y

$a = 9.81 m/s^2$ down

$V_{oy} = 0$

$V = ?$

$y = 62m$ down

$t = ?$

How high = 62m

Start w/ y-direction

Find time

$y = V_o t + \frac{1}{2} a t^2$

$62m = \frac{1}{2}(9.81) t^2$

$3.56s = t$

How much time

x-direction

$V = \frac{\Delta x}{t}$

$12 = \frac{x}{3.56}$

$x = 42.7m$

How far

Same problem continued

Another question to look at in this problem might be the velocity of the rock just before impact. We know it's x-velocity since that is constant. Need to find the y-velocity and use vector addition to combine them.

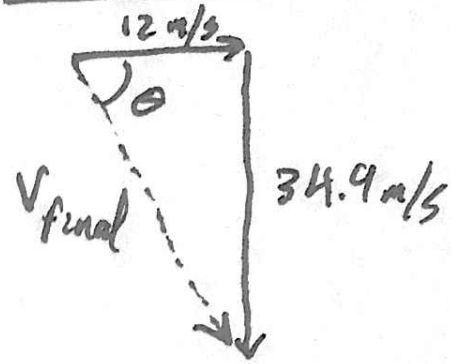
y-direction

$$a = \frac{\Delta V}{t}$$

$$9.81 = \frac{V - 0}{3.56}$$

$$34.9\ m/s = V$$

Combined



$V_{final} = 36.9\ m/s$ @ 71° into the ground

C) Finally, we come to the toughest of the three type of projectiles: Full Trajectory. The reason these are the toughest is because they require the most thought that is outside of the mathematics of solving the problem. Here are a few key ideas:

i: What goes up must come down. The distance up is the same distance down. The time it takes to go up is the time it takes to go down.

ii: The total time of flight is twice the time to go up, or twice the time to come down

iii: The vertical velocity of the object at the max height, or at the top of the projectile arc is zero. An object rising in the air must slow down and come to a stop before it begins falling. This key point it time will give you the extra information you need to solve these problems.

iv: all velocities must be resolved into either horizontal or vertical velocity vectors using tirgonometry

Let's play baseball!

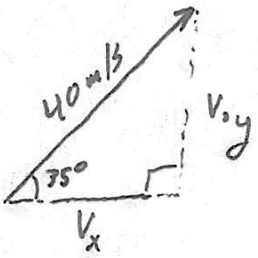
A batter hits a pop-up at 35 degrees upward with an initial speed of 40 m/s. How high does it go? How far away does it land? How long was it in the air?

Set-Up:

On next page

$V_0 = 40$ m/s @ $35°$ upwards

How high? How far? How much time?



$V_x = 40\cos 35° = 32.8$ m/s

$V_{0y} = 40\sin 35° = 22.9$ m/s

x

constant velocity

$x = ?$

$V = 32.8$ m/s

$t = ?$

$V = \frac{\Delta x}{t}$

can't start here, 2 unknowns

So go to y-direction

$V = \frac{\Delta x}{t}$

$32.8 = \frac{x}{4.67}$ ← total time

153m = x

y

$a = 9.81$ m/s^2 down

$V_{0y} = 22.9$ m/s up

$V = 0$ at top of trajectory

$y = ?$

$t = ?$ ← time up ($t_{\frac{1}{2}}$)

to find t

$a = \frac{\Delta V}{t}$

$-9.81 = \frac{0 - 22.9}{t_{\frac{1}{2}}}$

$t_{\frac{1}{2}} = 2.33$s ← time up

$t_{tot} = 4.67$s

to find height

$x = v_0 t + \frac{1}{2} a t^2$

$y = (22.9)(2.33) + \frac{1}{2}(-9.81)(2.33^2)$

y = 26.7m